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Spectral characteristics of some deep current records from the eastern North Atlantic

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A brief résumé is given of the work performed since 1967 by the National Institute of Oceanography in the collection of current meter records by the use of deep sea moorings and recording current meters. These records are, unlike most other current observations, accompanied by simultaneous observations of water temperature.

It is possible from a knowledge of the vertical profile of temperature at or near to a mooring to derive from the temperature observations a time series which approximates to that of the vertical movement of the water.

The object of this paper is to investigate the validity of equating the derived time series to that of the true vertical water motion. The spectra of potential energy of vertical motion derived from the temperature records are compared with the simultaneous spectra of horizontal kinetic energy derived from the measurements of horizontal velocities.

It is concluded that under circumstances in which there are well-defined vertical gradients of temperature and density the derivation of vertical motion from a temperature record is valid and can be used with a good degree of confidence to investigate the frequency distribution of potential energy in the inertial subrange (extending from the local inertial frequency to the Brunt–Väisälä stability frequency).

An idealized observational scheme is suggested which would in most cases fulfil the necessary requirements for a simple monitoring of vertical motions over long periods of time.

1. Introduction

Since 1967 the National Institute of Oceanography has collected a number of records of subsurface currents from sites in the eastern North Atlantic Ocean. The location of these sites is shown in figure 1 which illustrates their positions relative to the 200 m contour marking the edge of the continental shelf. All the sites are in water depths greater than 1000 m but site N is anomalous since it is a test site on a fan in the continental slope and has near to it water depths varying between 200 and 4000 m. Sites J and B are both in deep water, 3000 m at J and more than 4000 m at B. The records from site B have been summarized by Gould (1969) and it is intended to produce similar summaries for the data from the other sites.

The records were, in general, obtained from moorings with subsurface buoyancy using Bergen recording current meters. These instruments have been fully described by Aanderaa (1964) and to the present day have been modified only in non-essential details from the description given by Dahl (1969). The current data are recorded in digital form on 6 mm magnetic tape once every sample period. The parameters measured are current speed sensed by a rotor and integrated over the recording interval, current direction by measuring the orientation of the fin relative to magnetic north and water temperature by a thermistor projecting through the pressure case. The resolution of the speed sensor is variable, depending on the choice of gear ratio and sample interval. Directions are resolved to the nearest 0.35° and temperature to 0.025 °C, the thermal time constant of the temperature sensor is about 20 s.

In most cases the objective of the current measurements has been to study the variability over periods of weeks of the currents at a site. An example of this type of interpretation is given by Gould (1971) in which a change in the current pattern is detected by the use of progressive vector diagrams derived from the current record. Figure 2 shows a typical progressive vector diagram from site N. The record was taken at a depth of 350 m in a water depth of 2000 m and shows in addition to the usual semidiurnal fluctuations other abrupt changes in mean direction which may well be associated with the bottom topography near that site. Such a representation, which can loosely be compared with the trajectory of an imaginary particle of water, does not illustrate the distribution of energy in the various frequencies present in the motion. In order to do this techniques of spectral analysis are employed.

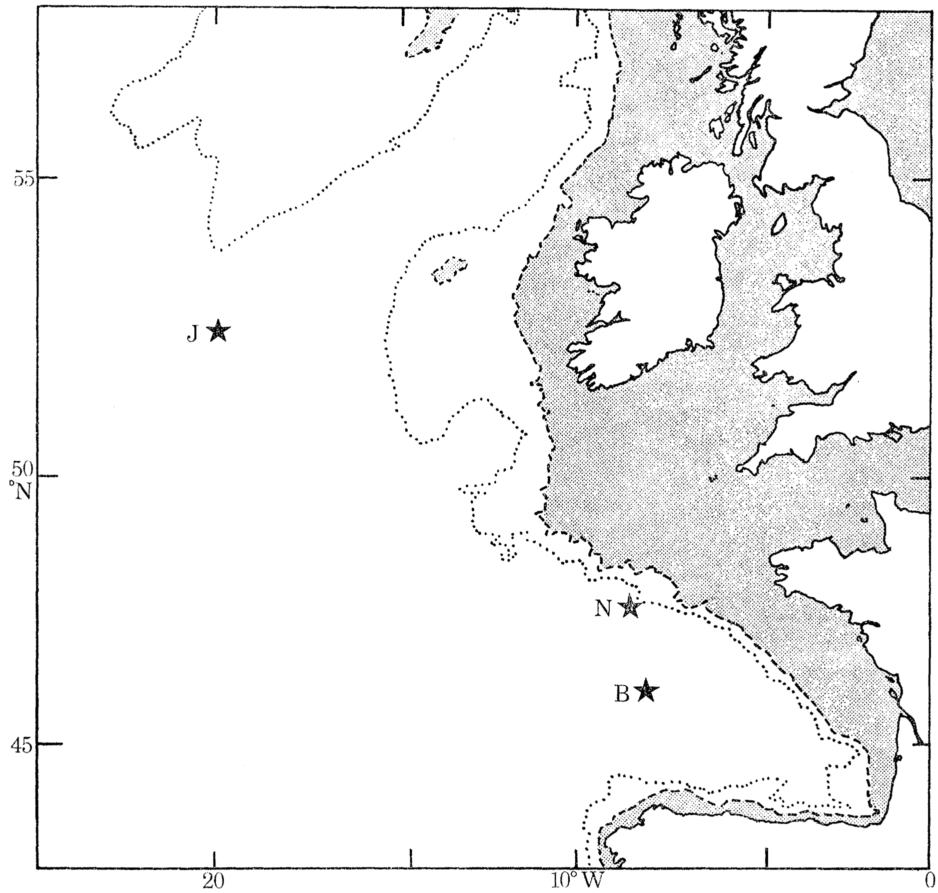

FIGURE 1. Chart of eastern North Atlantic Ocean showing positions of N.I.O. mooring sites. Stippled area shows extent of continental shelf. ----, approximate 200 m contour; ..., approximate 2000 m contour.

Two typical power spectra of horizontal currents are illustrated in figure 3. The two records used here are from site N at 350 m and from site B at 400 m. The former is the same record that was presented in figure 2. Spectra from these two sites are very similar to those from Woods Hole site D and exhibit high energy densities at the semi-diurnal tidal period and at the local inertial period (*ca.* 16.5 h) and an energy minimum around 25 to 30 h. At frequences above the semi-diurnal the energy falls with increasing frequency with a power law close to $-\frac{5}{3}$.

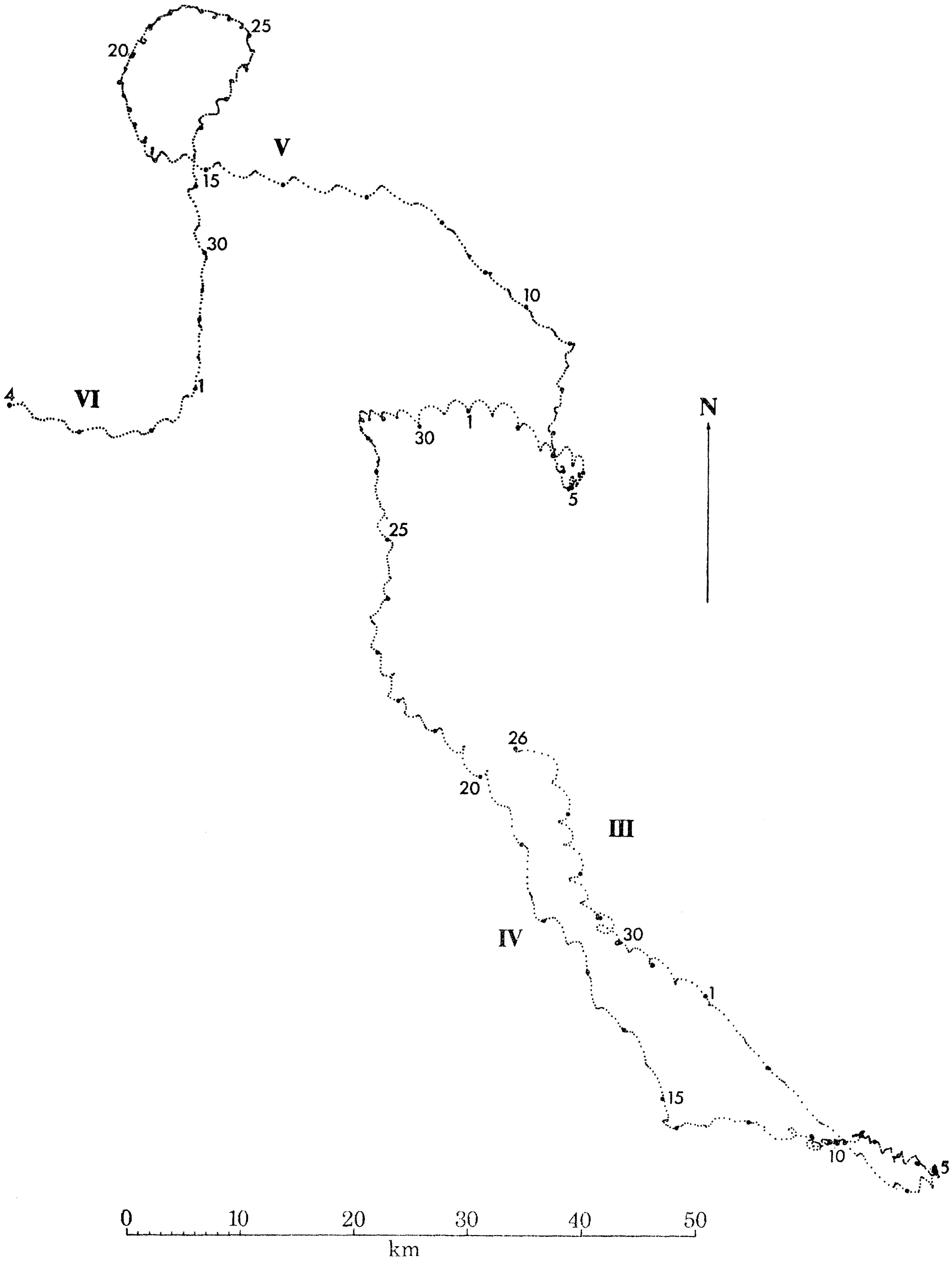

FIGURE 2. Progressive vector diagram of record 03901 from site N at 350 m. Points marked are at daily and hourly intervals. Roman numerals denote months in 1969.

2. The direct measurement of vertical motions in the oceans

The techniques mentioned above are typical of many which have been employed for several years for the measurement and analysis of the horizontal movements of water within the oceans. The measurement of vertical motions is very much more difficult and until recently had received little attention. In general the ocean is stably stratified and thus vertical movements of water are suppressed and involve velocities which are typically several orders of magnitude smaller than

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the horizontal velocities. This dominance by the horizontal motions presents very great obstacles to the direct measurement of vertical motion but direct measurements have been made by a method described fully by Webb, Dorson & Voorhis (1970).

The method involves the use of a neutrally buoyant float which sinks to a preset level and then follows the horizontal currents. Movements of water vertically past the float cause it to be turned

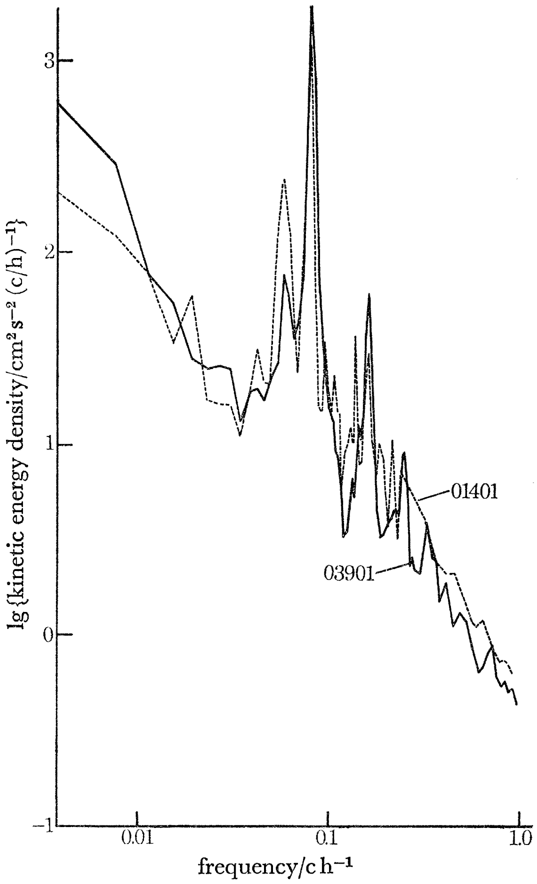

Figure 3. Spectra of horizontal kinetic energy for records 01401 (site B, 400 m) and 03901 (site N, 350 m).

by angled fins around its circumference and these rotations are recorded internally. Contamination of the record by the horizontal currents is avoided since the float moves with these currents. The method has been employed in several oceanographic situations and results have been published by Voorhis (1968), Webb & Worthington (1968) and Voorhis & Webb (1970). The method has one major disadvantage in that the vertical motions cannot be measured in one fixed point over a period of time since the sensor is constantly changing its horizontal position. There is, however, an indirect method of observation which can, under favourable circumstances, give reliable information concerning the vertical movements of water.

3. A possible indirect method of measuring vertical motions

This indirect method involves the continuous recording of some parameter which has a known vertical gradient. Temperature is such a parameter which in general has a measurable vertical gradient within the ocean and which is easily recorded. The process of deriving the vertical displacement from the temperature record is that of finding a solution $z = \xi$ at some time t satisfying the two equations $\theta = g(z,t)$ and $\theta = f(t)$ where $g(z,t)$ describes the vertical structure of temperature and $f(t)$ is the time series of temperature observed at a fixed point in space. In the case of a constant linear gradient of temperature this reduces to

$$\xi = \frac{\theta - \theta_0}{\mathrm{d}\theta/\mathrm{d}z}, \tag{3.1}$$

where θ_0 is the mean temperature at the depth of observations. Records of temperature obtained from moored buoys reveal well-marked periodicities and it seems appropriate to investigate the energy distribution in the motions by the techniques of spectral analysis.

The potential energy per unit volume for a vertical displacement is given by $\frac{1}{2}g\,\Delta\rho\xi$ where $\Delta\rho$ is the density change. Now $(g/\rho_0)\,(\mathrm{d}\rho/\mathrm{d}z) = N^2$ where N is the Brunt–Väisälä frequency.

Therefore the energy

$$= \tfrac{1}{2}\rho_0 N^2 \xi^2$$
$$= \tfrac{1}{2}(\rho_0 N^2/\omega_0)\int_0^\infty P_{\xi\xi}\,\mathrm{d}\omega,$$

where $P_{\xi\xi}$ is the spectral density of the vertical displacements in a frequency band ω_0.

Hence $\frac{1}{2}N^2 P_{\xi\xi}$ is the potential energy density in ergs per gramme per frequency interval ω_0 Using the assumption of a linear gradient of temperature the potential energy density can be derived directly from the temperature spectrum using the relation

$$P_{\xi\xi} = P_{\theta\theta}/(\mathrm{d}\theta/\mathrm{d}z)^2,$$

$$\text{Potential energy density} = \tfrac{1}{2}\{N^2/(\mathrm{d}\theta/\mathrm{d}z)^2\}P_{\theta\theta}.$$

The derivation of the spectral relationships between the horizontal, and vertical energy densities has been performed by Fofonoff (1969) and it is from this work that the above relationships have been taken.

4. Limitations of the indirect method

As has been mentioned in the previous section an assumption is made concerning the linearity and constancy of the vertical gradient of temperature. Cox (1968) has pointed out that failure to satisfy this assumption may have a serious effect on the derived time series and on the spectra of the vertical motions. Within the ocean the vertical structure of temperature consists of a series of more or less regularly spaced intervals of greater or lesser gradient about a mean value. This microstructure can exist on vertical scales varying from a few centimetres up to several metres and in Cox's view such structure invalidates the use of equation (3.1) for all but the dominant periodicity.

It is possible to see the effect of an idealized microstructure on a pure sinusoid in the vertical motion. If the vertical structure of temperature obeys the relation

$$\theta = \theta_0 + \alpha z + \tau_0 \sin(2\pi z/h)$$

and the water moves vertically with amplitude, A_0 and frequency ω according to $z = A_0 \sin \omega t$.

Then the time series of temperature θ recorded at some level will be of the form

$$\theta = \theta_0 + \alpha A_0 \sin(\omega t) + \tau_0 \sin\left\{2\pi\left(\frac{A_0}{h}\sin \omega t\right)\right\}.$$

The first two terms represent a pure sinusoid which is the ideal output and the last term gives the disturbance due to the microstructure. This is a signal of amplitude τ_0 but with a frequency which varies between $\omega(1+A_0/h)$ and ω. Thus the deviation from the true response is an addition of energy in a frequency band with a lower limit at the frequency of the pure motion and a high-frequency limit dependent on the vertical scale of the dominant microstructure. The amplitude of the deviation from the true response is constant and equal to the microstructure amplitude. Since the frequency of the microstructure-caused component is changing due to the changing vertical velocities involved the distribution of energy with frequency is such that the energy is at

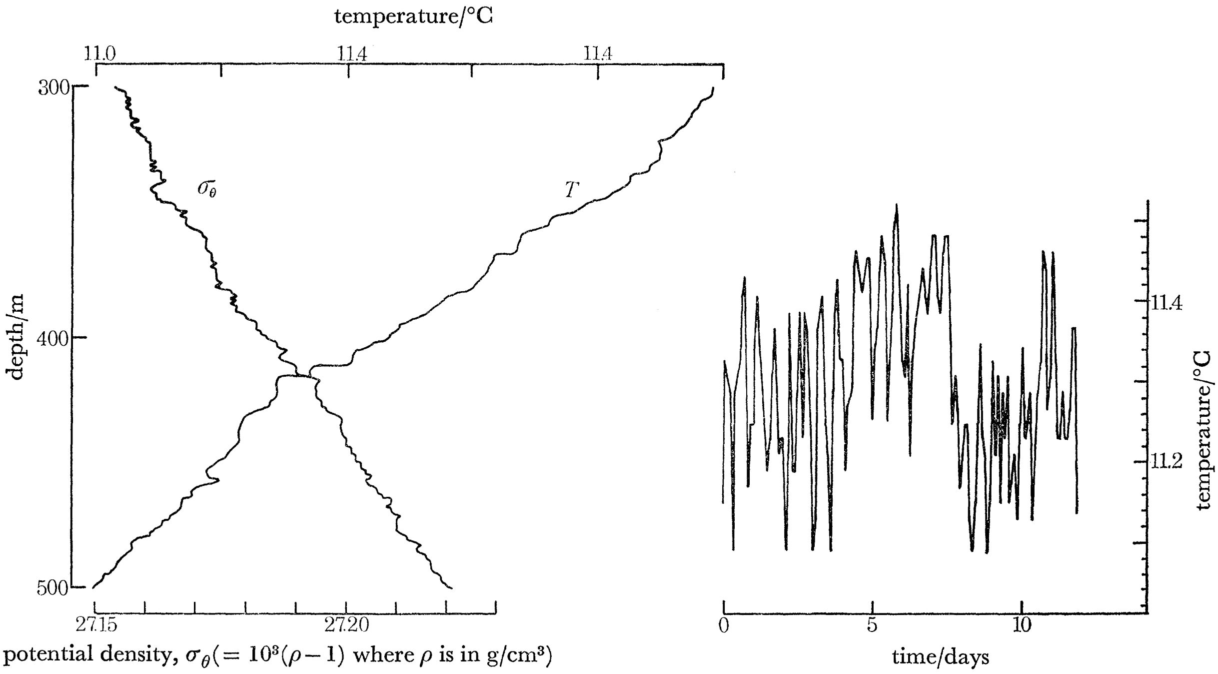

FIGURE 4. Vertical profiles of temperature and potential density for Discovery Hytech s.t.d. Stn 6535 (46° 00′ 3 N, 08° 21′ 0 W) together with section of 2-hourly mean temperature record 01401 (site B). The temperature scale on the time series has been adjusted so that the depth scale is correct for vertical motions.

a maximum at the higher frequency and falls to approach zero at the lower frequency. Hence the overall effect is to introduce energy at frequencies slightly below $\omega(1+A_0/h)$.

Consider the situation at site B at a depth of around 400 m. Figure 4 shows a sample of the temperature record from that position and depth together with the vertical profiles of temperature and density over the depth range 300 to 500 m. The temperature record is made up from 2-hourly mean values of temperature derived from the original time series of 10 min values and shows a dominant semi-diurnal motion with a peak to peak amplitude of around 0.2 °C. The mean temperature gradient is around 0.25 °C/100 m, and thus the semi-diurnal motion has a peak to peak amplitude of 80 m. In terms of the idealized structure mentioned previously $h \approx 10$ m and $\tau \approx 0.01$ °C.

The ratio of the energy of the vertical semi-diurnal motions to the maximum energy in the components due to microstructure, which in this case would have a peak at around 2.5 h will be around 1:500. This means that the energy which is observed at that frequency in records from 400 m and which is only a factor of 100 less than the vertical semi-diurnal peak is probably real. It is certainly possible that in some cases large disturbances from a smooth mean profile of temperature can be introduced by the advection of water with appreciably different characteristics but these phenomena tend to be intermittent in nature and contribute little to the overall energy spectrum.

It has been mentioned in § 1 that the time constant of the thermistor of the current meter is about 10 to 20 s. The thermistor response is not simple since the thermistor body which has a small heat capacity is not thermally insulated from the massive pressure case, this results in an initially fast response (time constant 7 s) and then a much slower response (time constant about 50 s). In view of the resolution of the recording system, the relation of the recorded output to the true temperature signal depends on the form of the signal. A worst case can be considered using the 50 s time constant and this shows that for an oscillation of only 5 min period the energy recorded would be 97 % of the true energy. In the cases considered here no appreciable error will be introduced by the thermistor time constant.

The fact that the observations are made at a fixed point in the ocean has one disadvantage regarding the interpretation of specific peaks in the spectra investigated. This is due to the Doppler effect on the frequencies of the wave motions. Water is continuously being advected past the mooring and the water velocity modifies the frequencies of processes occurring with the water.

The features which the model outlined in the next section sets out to investigate are the progressive internal waves and these typically have low velocities of propagation. Because of this there are likely to be large frequency distortions which may act to either increase or decrease the true frequency depending on the relative directions of the mean advective velocity and the velocity of propagation. Although there may be frequency shifts these will be of the same magnitude and in the same sense for the horizontal and vertical motions and so although the absolute values of energy may be in error the ratio of the energies in the vertical and horizontal motions will not be affected.

A problem occurs in the interpretation of events at frequencies near to the Brunt–Väisälä frequency. It is thought that stability oscillations break up into a cellular formation which again would be advected past the mooring. In this case the frequency shift would always be so as to increase the true frequency but the magnitude of the shift would be dependent on the size of the cells. If the cells were of such a size that their horizontal length scale were greater than the product of the mean advective velocity and the Brunt–Väisälä period then there would be no detectable shift in frequency. This length scale for the data considered is of the order of 300 m. It appears that in many spectra there is a detectable drop in energy near to the observed Brunt–Väisälä frequency which tends to support the idea that the cells have a horizontal size greater than this 300 m value.

A halving of the horizontal length scale would double the apparent Brunt–Väisälä frequency. In all cases where a detectable drop in energy occurs the frequency is within 10 % of the observed Brunt–Väisälä frequency suggesting that if there is a cellular structure it is certainly of a size greater than 300 m.

5. The energy ratios for an internal wave system

Fofonoff (1969) has derived relationships between the spectral functions of potential energy and both horizontal and total kinetic energy for an idealized linear internal wave system within the inertial subrange.

His results are

$$\frac{\text{potential energy}}{\text{horizontal kinetic energy}} = \frac{N^2 P_{\xi\xi}}{P_{uu}+P_{vv}} = \frac{N^2}{N^2-\omega^2}\left(\frac{\omega^2-f^2}{\omega^2+f^2}\right), \tag{5.1}$$

$$\frac{\text{potential energy}}{\text{total kinetic energy}} = \frac{N^2 P_{\xi\xi}}{P_{uu}+P_{vv}+P_{ww}} = \frac{N^2}{N^2-\omega^2}\,\frac{\omega^2-f^2}{(\omega^2+f^2)+\omega^2(\omega^2-f^2)}, \tag{5.2}$$

where ω is a frequency in the inertial subrange and f is the local inertial frequency.

The form of these ratios is, in the case of (5.1), an increase from zero at f through a plateau where the ratio is close to unity and then tending to infinity on approaching N. Equation (5.1) gives a similar result but the plateau continues to approach unity instead of infinity near to N.

Fofonoff presents data obtained from the floats described in § 2 in the form of a comparison of the spectral energies with those of horizontal motions recorded nearby at Woods Hole site D. The fit of the data to the theoretical curves is not particularly close and is subject to uncertainties due to the varying horizontal separation of the points at which the vertical and horizontal motions were sensed. However, the data show an 'order of magnitude' agreement between the energy levels.

6. Hydrographic data from site B

It has been shown that in order to derive a potential energy spectrum from the spectrum of temperatures, a knowledge is required of the values of the Brunt–Väisälä frequency and of the vertical gradient of temperature. Although moorings have been laid by the National Institute of Oceanography at three sites it seems from the cautions of Cox that quite detailed hydrographic data are required for a full analysis of the temperature data. It is only at site B that such data are available concurrent with the current and temperature records.

The current observations from site B were collected on a cruise in the autumn of 1967 and were accompanied by simultaneous vertical profiles taken both with water bottles and with Bathysonde and Hytech s.t.d. continuous profiling devices (Pingree 1969, 1970; Gould 1971). During a 2-month period a total of 11 Hytech s.t.d. and 20 Bathysonde stations were worked within a radius of approximately 35 km from the mooring positions. When scattered over such a large area these data give a rather inadequate picture of the hydrographic situation, especially in view of the presence of a feature which was observed to move through the area (Gould 1971). The data do however give a feeling for the variability of the temperature gradient and Brunt–Väisälä frequency at the depths of 400 and 1400 m from which current records were obtained.

A sufficiently detailed analysis to resolve the microstructure scales in every record would be extremely laborious and so mean values of temperature gradient and Brunt–Väisälä frequency were computed for each record over depth intervals of 100 and 200 m about the central depth. This is considerably better than a 'climatic' mean value for the area taken from surveys at various times over a period of years.

The structure of the water column in the Bay of Biscay is rather complex and has as its main feature the intrusion of a core of warm, high salinity water of Mediterranean origin at a depth of 1000 m. This core of water has the effect of dividing the water column into two parts, above the core a region of small temperature gradients and a stabilizing salinity gradient and below the core very strong stabilizing temperature gradient and highly destabilizing salinity gradients. The core itself is marked by a salinity maximum and extreme variability in both temperature and salinity. The 400 m level is in the region of small temperature gradients and the 1400 m level in the large gradient zone but below the region of maximum variability; in spite of this, instrusions of anomalous water do appear from time to time, in the depth interval used around the 1400 m level.

The temperatures and salinities were computed from the recorded frequency outputs from the Hytech s.t.d. and for the Bathysonde from a small-scale $X-Y$ plot of the temperatures and conductivities. The use of such small scale plots and the fact that in many cases a range-change occurred at around 1400 m is likely to have introduced some errors to the Bathysonde outputs.

The temperature gradient values obtained were as shown in table 1.

The agreement at the 400 m level is good and all values have a good overlap in view of their standard deviations.

Table 1. Mean temperature gradient values site B

depth interval	400 m		1400 m	
m	Hytech s.t.d. °C/100 m	Bathysonde °C/100 m	Hytech s.t.d. °C/100 m	Bathysonde °C/100 m
100	0.255 ± 0.034	0.259 ± 0.083	0.986 ± 0.142	0.800 ± 0.138
200	0.245 ± 0.021	0.261 ± 0.053	0.949 ± 0.098	0.793 ± 0.115

Table 2. Mean values of Brunt–Väisälä frequency site B

depth interval	400 m		1400 m	
m	Hytech s.t.d. c/h	Bathysonde c/h	Hytech s.t.d. c/h	Bathysonde c/h
	1.212†	1.340	1.747	1.769
100	1.113	1.176	1.600	1.507
	1.003	0.972	1.453	1.187
	1.132	1.287	1.743	1.701
200	1.095	1.187	1.608	1.574
	1.057	1.077	1.463	1.436

† The limiting values are both given since the values of Brunt–Väisälä frequency were computed from the mean and standard deviations of the density gradients and, as such, the process of taking the square root of these values introduces some distortion.

At 1400 m there are significant differences between the values computed from the two different instruments. Although the values do overlap the cause of the disagreement may not be purely instrumental. The Bathysonde was used throughout the cruise but the Hytech s.t.d. stations were only occupied during the second of 2 months. This means that any time dependent effects with periods greater than 2 months would produce differences between the two instruments. Another factor is that the area of operation moved somewhat to the northeast during the latter part of the cruise and this may introduce other differences.

The values for Brunt–Väisälä period are given in table 2.

The large range of values on the Bathysonde is probably due mainly to the errors introduced by the range change and by reading from small-scale plots. Accepted values for the area used in the evaluation of potential energy spectra were as in table 3.

These represent values biased in favour of the early Bathysonde results since the current records were all taken during the first half of the cruise in positions near to where the early stations were occupied.

The results demonstrate that even within a relatively small area there can be considerable

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variability in the mean values of temperature and density gradients over depths of 100 m or more. The values computed over smaller depth intervals exhibit increasing variability as the vertical scale of the microstructure is approached.

The station data give far from ideal information and for adequate coverage a time series of these gradients at a fixed point near the mooring position would be needed.

Table 3. Accepted values of temperature gradient and Brunt–Väisälä frequency for site B

level/m	400	1400
temperature gradient/°C $(100\,\mathrm{m})^{-1}$	0.255	0.900
Brunt–Väisälä frequency/c h^{-1}	1.100	1.400

7. The spectra of temperature

Figure 5 shows a typical temperature spectrum from 1400 m.

The two main features are high energy near the semidiurnal tidal frequency and a sharp drop in energy close to the Brunt–Väisälä frequency. The spectrum shown is a composite of two with the low-frequency section from a spectrum with few degrees of freedom and the high-frequency part with narrower confidence limits. The spectrum shows also a general noisiness at frequencies

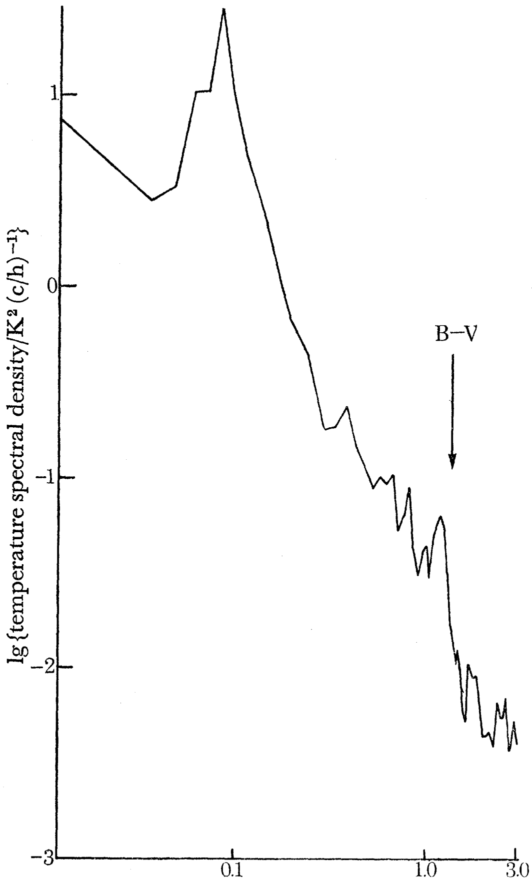

Figure 5. Spectrum of temperatures from record 01302 (site B, 1400 m). Arrow marks the local Brunt–Väisälä frequency.

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immediately below the Brunt–Väisälä frequency. This may well be caused by interaction of contributions from both microstructure and true vertical water movements but occurs at frequencies higher than would be expected from the arguments outlined in § 4. The marked drop in energy level near the Brunt–Väisälä frequency has been noted by Voorhis (1968) from the direct observation of vertical motions.

A general feature of the temperature spectra is that in the longer records from both 400 and 1400 m the drop in energy at the Brunt–Väisälä frequency is much less marked. This seems to have a possible explanation in the variability of Brunt–Väisälä frequency with time which would smooth out the sharp drop. The Nyquist frequency for these longer records may well not be high enough to see the cutoff which should still occur at the highest Brunt–Väisälä frequency experienced.

The low energy levels above the Brunt–Väisälä frequency imply that aliasing is not likely to cause a gross folding of energy from the high-frequency motions.

8. The analysis of potential energy

The temperature spectra for several records were converted to potential energy spectra by the relationship

$$PE = \tfrac{1}{2}N^2 P_{\theta\theta}/(\mathrm{d}\theta/\mathrm{d}z)^2$$

and the spectra thus obtained were compared with the spectra of horizontal kinetic energy from the current meter records. One shortcoming of the current meter data from the site B records is that owing to the lack of low ratio gearboxes a large digitizing interval had to be tolerated for the speeds. This leads to a high noise level which affects the spectra at frequencies above 1 c/h and invalidates any comparison above this frequency. The values of $\mathrm{d}\theta/\mathrm{d}z$ and N used are the accepted values given in table 3.

The features mentioned for the temperature spectra are exactly reflected in the potential energy spectra since only a constant factor is used in the conversion.

Figure 6 shows a comparison between the spectra of potential energy and horizontal kinetic energy for a record from 1400 m depth. It illustrates the very close agreement between the two spectra over almost the whole frequency range and certainly the values lie well within the ranges set by the confidence limits. In addition, a spectrum of the vertical kinetic energy is calculated by multiplying each potential energy estimate by ω^2/N^2. This spectrum shows remarkable similarities to those given by Voorhis (1968) with low energies at low frequencies, rising to a plateau at frequencies between the semi-diurnal and just below the Brunt–Väisälä and then falling steeply.

Figure 7 shows another spectrum, this time for a long time series at 400 m. In this record the agreement between the potential energy and the horizontal kinetic energy is not as good and barely falls within the confidence limits. Due to the absence of the sharp drop at the Brunt–Väisälä frequency that was mentioned in the previous section the plateau is not properly defined and has no high-frequency limit.

Another way of looking at the data is to take the ratios between the potential energy estimates and the estimates of both horizontal and total kinetic energy, the latter being computed from the sum of the horizontal and vertical energies. The first ratio plot (figure 8) is for the data of the previous diagram and shows an excess of potential energy over kinetic energy at all frequencies except near 1 c/h, where the horizontal kinetic energy is expected to be too high due to the digitizing interval.

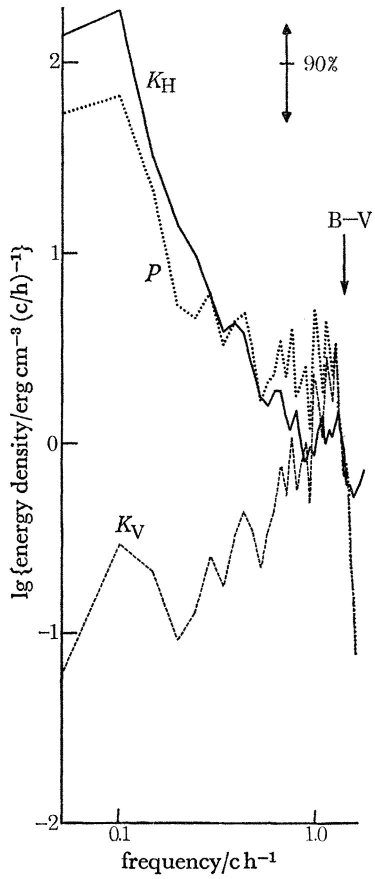

FIGURE 6. Spectra of horizontal kinetic energy (K_H) vertical potential energy (P) and vertical kinetic energy (K_V) for record 01403 (site B, 1400 m).

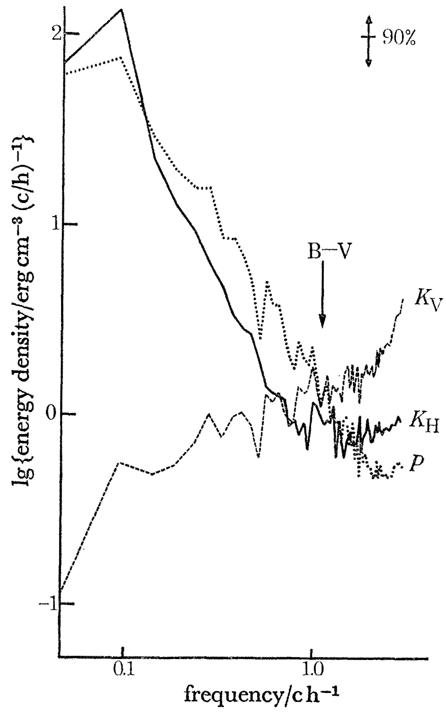

FIGURE 7. Spectra of horizontal kinetic energy, vertical potential energy and vertical kinetic energy for record 01401 (site B, 400 m).

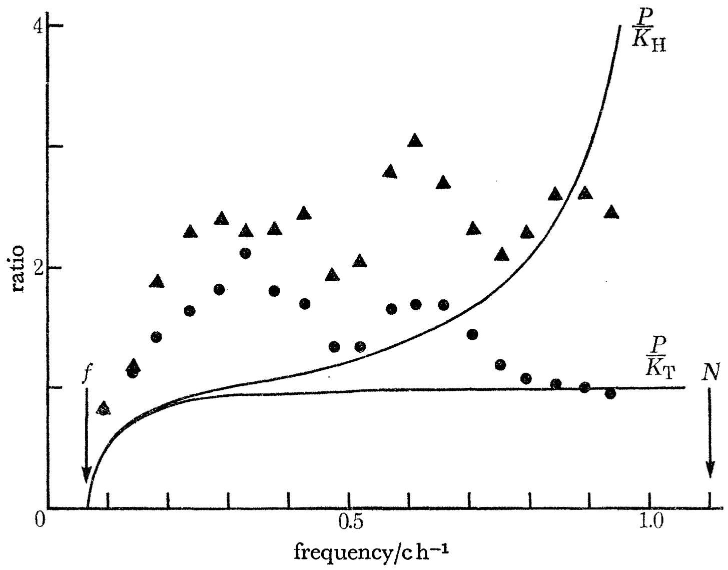

FIGURE 8. Ratio of vertical potential energy to horizontal and total kinetic energy. Theoretical curves from equations (5.1) and (5.2) are shown together with experimental points. N marks the Brunt–Väisälä frequency, f marks local inertial frequency.

Experimental points from record 01401 (site B, 400 m). ▲, ratio of potential energy density to horizontal kinetic energy; ●, ratio of potential energy density to total kinetic energy.

The second (figure 9), shows a different case in which the kinetic energy is too high. In the third case shown (figure 10) the agreement with the theoretical curves is extremely good.

The accepted values for the temperature gradient and Brunt–Väisälä frequency were derived from rather inadequate data. The comparison of the ratios of the vertical potential energies with total and horizontal kinetic energies show either uniformly good or uniformly bad fit to the theoretical curves over the whole frequency range. There are cases where the potential energy

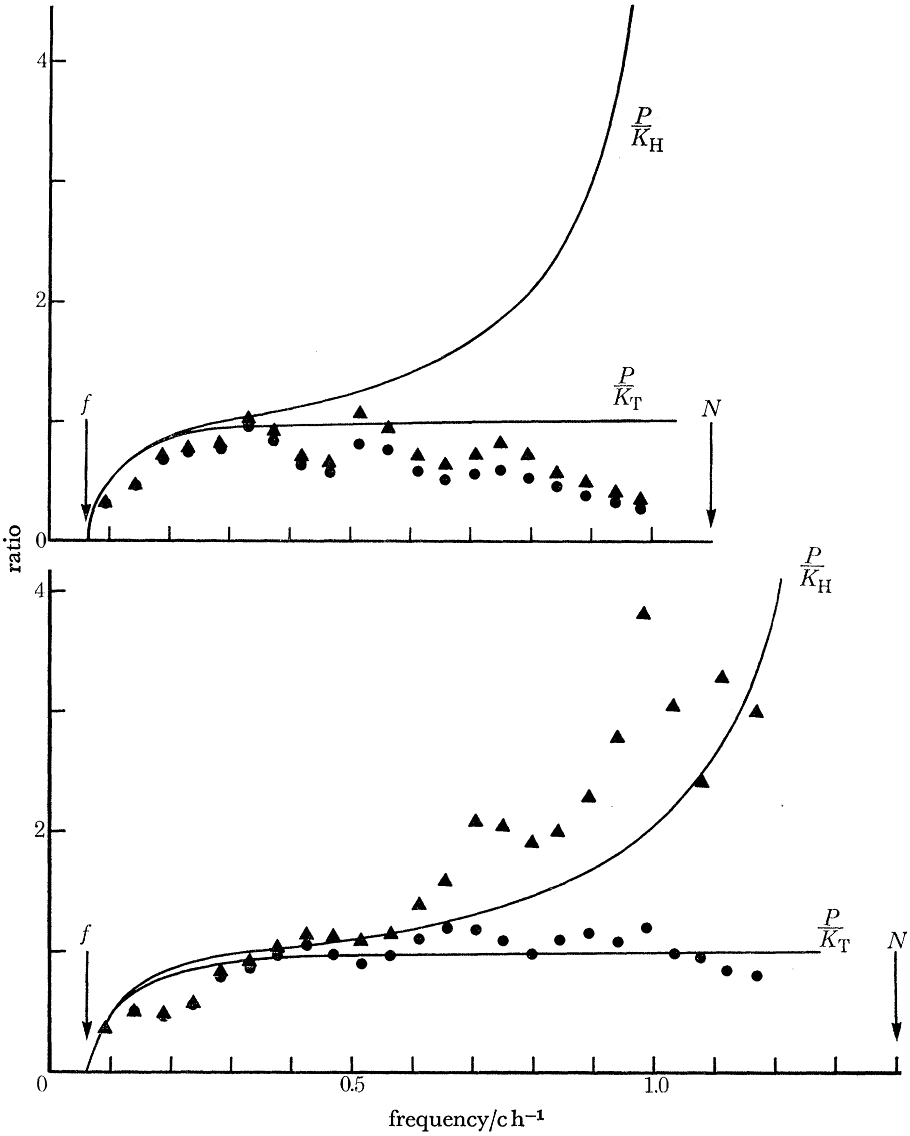

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FIGURE 9. Ratio of vertical potential energy to horizontal and total kinetic energy. Theoretical curves from equations (5.1) and (5.2) are shown together with experimental points. N marks the Brunt–Väisälä frequency, f marks the local inertial frequency.

Experimental points from record 01201 (site B, 400 m). ▲, ratio of potential energy density to horizontal kinetic energy; ●, ratio of potential energy density to total kinetic energy.

FIGURE 10. Ratio of vertical potential energy to horizontal and total kinetic energy. Theoretical curves for equations (5.1) and (5.2) are shown together with experimental points. N marks the Brunt–Väisälä frequency, f marks the local inertial frequency.

Experimental points from record 01403 (site B, 1400 m). ▲, ratio of potential energy density to horizontal kinetic energy; ●, ratio of potential energy density to total kinetic energy.

is too great and also ones where the potential energy is too small over the whole frequency range. This throws into serious doubt the validity of the accepted values of the temperature gradient and Brunt–Väisälä frequency.

If microstructure-caused energy were a significant factor it would be expected that the spectral ratios would tend to have a maximum value at the frequencies at which this energy should appear. No such peaks in the spectral ratios are seen and hence it is taken that microstructure had no appreciable effect in this situation.

The fact that tables 1 and 2 show large variations in both temperature gradient and Brunt–Väisälä frequency would naturally lead one to treat any mean value with some caution. Since both of these parameters are squared before use in the derivation of potential energy for each parameter the range of values is capable of introducing a factor of two in the spectral ratios.

In view of this it is impossible to draw any conclusions about the validity of the linear internal wave model. Certainly there is agreement within an order of magnitude such as was noted by Fofonoff (1969), but for an adequate test of the model much more comprehensive data covering the variability with time of the gradients of temperature and density are required.

9. An idealized scheme of measurement

Since it seems that the evaluation of the temperature gradient and Brunt–Väisälä frequency were inadequate in the data presented in the previous section it is worth while to attempt to outline an idealized observational scheme which would overcome these difficulties.

Temperature gradient is relatively easy to measure with the accuracy required for this type of work. A minimum requirement is the sampling of temperature gradient over the maximum extent of the vertical motions at the same times at which horizontal currents are recorded. A better scheme is one in which the temperatures are recorded at a series of depths throughout the vertical extent of the motion to give some indication of the linearity of the gradients.

The derivation of a time series of Brunt–Väisälä frequency is much more difficult since *in situ* recorders for temperature and salinity suitable for use on moored buoys do not yet have the required accuracy to make the derivation of density values possible. This seems to be the prime requirement and until such time as these recorders or sensors to measure density directly become available it would seem that the Hytech s.t.d. probe used sufficiently frequently could give a reasonable indication of variability of Brunt–Väisälä frequency and in addition a monitor of the nature of the microstructure.

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